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CALIBRATION OF NUMERICAL MODELING AND A NEW DIRECT METHOD FOR CALCULATION OF SHALLOW FOUNDATION SETTLEMENTS IN SAND

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Abstract: Nonlinear methods for calculation of shallow foundation settlements in sand are addressed in the paper. One of them relies on the Hardening Soil Small constitutive relationship incorporated in the computer program Plaxis 2D. It has a significant drawback in the need to model the unknown strain history of the foundation soil prior to simulating the load test on a shallow foundation. This can be overcome by preloading the soil with a pressure which gives good matching between calculated and measured settlements. However, predictions of soil settlements cannot be made in such a way, because the soil strain history is not a priori known. It is shown in the paper how predictions of soil settlements can be made by Plaxis 2D by calibrating calculations against a simple direct method. A new direct method is developed along the lines of this simple method, on the basis of results of 16 load tests performed at 4 locations. The important advantage of the new method is that it properly takes into account the soil behavior at very small strains.

Keywords: shallow foundations, settlements, soil stiffness, initial shear modulus, results of load tests, new method for settlement calculation, very small strains

Sažetak: U radu se razmatraju nelinearne metode za proračun slijeganja plitkih temelja u pijesku. Jedna od njih se oslanja na konstitutivni odnos Hardening Soil Small uključen u računalni program Plaxis 2D. Ona ima značajan nedostatak u potrebi modeliranja nepoznate povijesti deformacije temeljnog tla prije simuliranja pokusnog opterećenja na plitkom temelju. To se može prevladati predopterećenjem tla tlakom koji daje dobro slaganje između izračunatih i izmjerenih slijeganja. Međutim, na takav način nije moguće napraviti predviđanja slijeganja tla jer povijest deformacije tla nije unaprijed poznata. U radu se pokazuje kako se Plaxisom 2D mogu napraviti predviđanja slijeganja tla kalibracijom proračuna prema jednostavnoj izravnoj metodi. U skladu s ovom jednostavnom metodom razvijena je nova izravna metoda na temelju rezultata 16 pokusnih opterećenja izvedenih na 4 lokacije. Važna prednost ove nove metode je to što na odgovarajući način uzima u obzir ponašanje tla pri vrlo malim deformacijama.

Ključne riječi: plitki temelji, slijeganja, krutost tla, početni modul smicanja, rezultati pokusnog opterećenja, nova metoda proračuna slijeganja, vrlo male deformacije



1. Introduction

The calculation of shallow foundation settlements is commonly based on the theory of elasticity and there are many methods which use correlations for elasticity parameters with results of in situ penetration tests (Standard Penetration Test – SPT, Cone Penetration Test – CPT). The foundation soil is assumed to behave linearly elastic. However, numerous laboratory tests and field load tests show nonlinear elasto-plastic soil behavior.

Benz (2007) presented a new constitutive relationship for soils, Hardening Soil Small (HSSmall), taking into account the shear modulus reduction from its maximum value at very small strains, and then nonlinearly decreasing with increasing shear strain. This constitutive relationship was incorporated into the Finite Element computer program Plaxis 2D (Brinkgreve, 2011). In the numerical analysis the experimental square footing is modeled as an equivalent circular footing of equal plan area. It will be shown in the next Section that even though the HSSmall relationship gives very good matching of the calculated load – settlement curve of a footing in sand, and measurements, the matching depends on the assumed soil strain history, which can be reconstructed in the numerical analysis by trial preloading of the foundation soil. The value of preloading is, thus, not known prior to a load test. Moreover, the HSSmall relationship requires multiple soil parameters determined from laboratory tests on undisturbed soil samples, which are very hard to obtain from coarse grained soils.

However, in cases of large footings, say, 30 m wide, and layered foundation soil, existing nonlinear methods for calculation of shallow foundation settlements cannot be used, so that numerical modeling is a viable option. It is shown in the paper how settlement predictions can acceptably be made by numerical modeling in such a way that the soil strain history is reconstructed by calibrating calculations against a very simple direct method for settlement calculation (Mayne et al., 2012). The only parameter required for the Mayne et al. (2012) method is the average CPT cone resistance of the foundation soil.

Direct methods for calculation of shallow foundation settlements comprise Mayne (2000), Akbas and Kulhawy (2009a), and Mayne et al. (2012). Mayne (2000) introduced a nonlinear relationship between settlement and average pressure the foundation exerts on soil, based on the linear expression proposed by Mayne and Poulos (1999), by using the Fahey and Carter (1993) modified hyperbola for the stress – strain relationship. This relationship describes the laboratory shear modulus reduction curve with increasing shear strain, and uses two parameters which have to be determined from triaxial or torsional shear tests. Mayne (2000) extrapolates this relationship to in situ conditions by using the ratio of the applied foundation pressure and soil bearing capacity instead of the ratio of laboratory deviatoric stress and deviatoric stress at failure. He also sets specific values for the two parameters, and verifies his method on one footing in sand and another in clay, with very good results. This method takes into account the soil behavior at very small strains, but requires the determination of the soil bearing capacity, for example by the Vesić method (Vesić, 1975).

Akbas and Kulhawy (2009a) proposed an empirical hyperbolic relationship between the ratio of the applied load and the limit load, and the ratio s/B , where s is the settlement and B is the size of the equivalent square footing, based on the L_1 - L_2 method (Hirany and Kulhawy, 1989), and by analyzing load – settlement measurements from 167 load tests at 37 locations. From these measurements Akbas and Kulhawy (2009a) define loads Q_{L1} and Q_{L2} which correspond to the end of the first linear part of the load – settlement curve and the beginning of its end linear part respectively, fitting the hyperbola between these two points for each footing. Q_{L2} corresponds to the limit load which can be determined based on the Vesić method for calculating the soil bearing capacity (Akbas and Kulhawy, 2009b). The authors (2009a) state that Q_{L2} is reached at $s/B = 0.539$.



Mayne et al. (2012) define the limit pressure at $s/B = 0.1$. It is commonly assumed in practice that soil failure occurs at this value of s/B . They proposed a very simple empirical relationship between the applied footing pressure and $(s/B)^{0.5}$ as a correlation with the CPT cone resistance, by analyzing measurements from 31 load tests at 13 locations. As much as it is very simple and useful for practice, the Mayne et al. (2012) method cannot predict the laboratory and field evidenced soil behavior at very small strains, which is important for seismic analyses of shallow foundations. Very small shear strains are between 10^{-5} and 10^{-4} (e.g. Lee et al. 2004). In this range of strains the soil stiffness is infinite according to Mayne et al. (2012) (and also according to Akbas and Kulhawy, 2009a), instead of being equal to the soil stiffness at very small strains (e.g. Burland, 1989). The shear modulus at very small strains can easily be determined from in situ measurements of the shear wave velocity.

Thus, a new direct method for calculation of shallow foundation settlements is presented in the paper. It is based on the Mayne et al. (2012) method, with elements of the Mayne and Poulos (1999) method, and an addition for properly predicting soil behavior at very small strains.

2. Calibration of numerical modeling

2.1. Test site at the University A&M Texas

Five load tests were performed at the University A&M Texas on square footings ranging in size from 1 m to 3 m, embedded 0.76 m into the 11 m thick layer of medium dense, fairly uniform silty fine silica sand, underlain by stiff clay (Briaud and Gibbens, 1994; 1997). Extensive in situ and laboratory tests were performed on the sand, and settlements were measured throughout load tests (Briaud and Gibbens, 1997). Among other sites, Benz (2007) simulated one of these load tests. He determined numerous soil parameters required for the HSSmall relationship from results of triaxial and resonant column tests. Benz published two sets of parameters, which differ only in values of the initial soil stiffness E_0^{ref} , and the shear strain $\gamma_{0.7}$ at which the secant shear modulus decreases at about 70% of its initial value (Table 1). The well known relationship between Young's modulus and the shear modulus is $E = 2G(1+\nu)$, where ν is Poisson's ratio. In situ Young's modulus at very small strains and confining pressure of 100 kPa, E_0^{ref} was determined from measurements of shear wave velocity, v_s from $G_0 = \rho v_s^2$, where ρ is the soil density.

Table 1. Two parameter values for HSSmall model (from Benz, 2007)

Parameter	Unit	Lab.	In Situ
E_0^{ref}	[MN/m ²]	260	390
$\gamma_{0.7}$	[-]	0.0002	0.0004

The difference between using the soil stiffness E_0^{ref} from laboratory and from in situ tests for numerical modeling is illustrated in Fig. 1 for the normally consolidated soil, which means that no strain history was simulated. Special attention has to be given to great discrepancies between measured and calculated settlements in the range of small strains for both sets of parameters. It can be said that in situ parameters work better. Besides, it is much easier to determine the soil stiffness at very small strains by in situ tests than it is by laboratory tests.

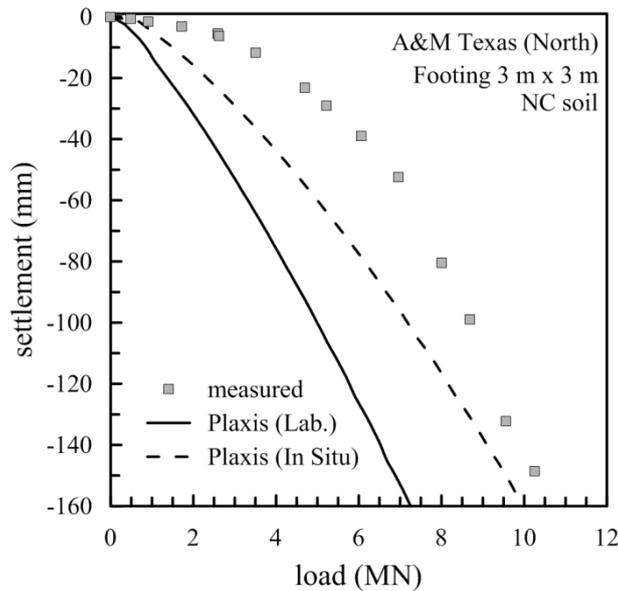
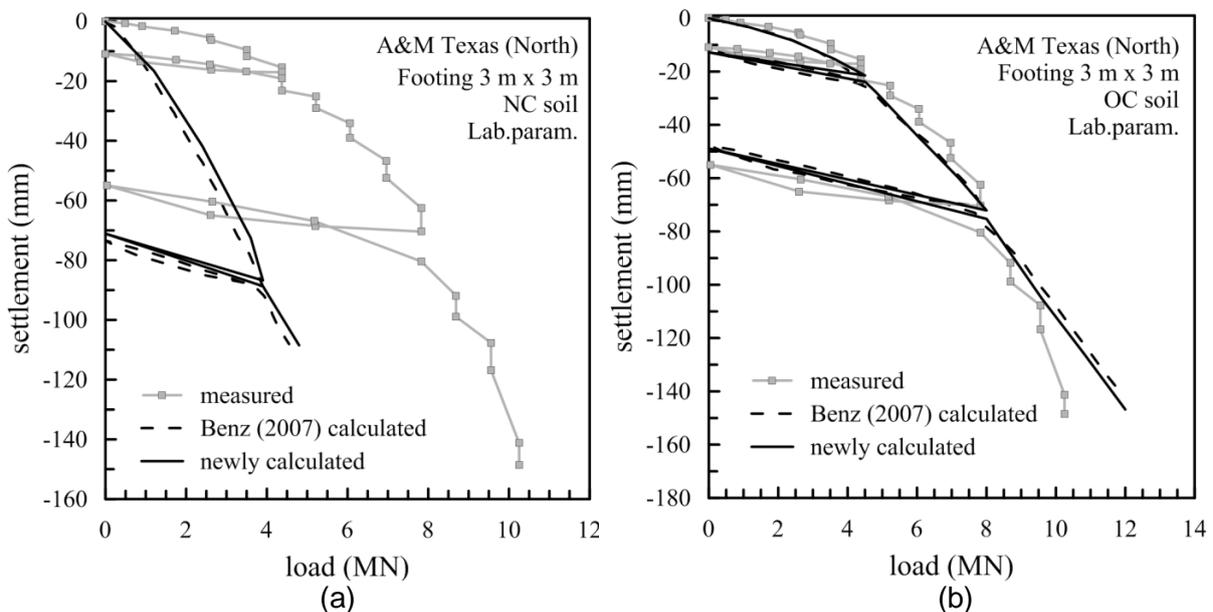


Figure 1. Measured and calculated settlements of A&M Texas 3 m x 3 m North footing; calculated settlements are with laboratory and in situ parameters from Table 1 for normally consolidated soil; measured settlements from Briaud and Gibbens (1997)

Benz (2007) shows 3 diagrams with calculated load – settlement curves for the selected footing. The first diagram is for normally consolidated soil and laboratory parameters. The second is for overconsolidated soil with the strain history reconstructed through preloading the foundation soil, and laboratory parameters. The third curve is for overconsolidated soil and in situ parameters. Benz (2007) does not state the values of required preloading pressures in the two cases of overconsolidated soil. Thus, the Authors newly calculated load – settlement curves in order to determine by trial the values of preloading necessary to match Benz’s curves (Fig. 2). The OC soil model was preloaded and unloaded, and the load test was then simulated. With laboratory parameters (Fig. 2b), the soil had to be preloaded by 360 kPa for the newly calculated curve. With in situ parameters (Fig. 2c), the soil had to be preloaded by 125 kPa. In situ parameters were used for further numerical analyses of A&M Texas footings.



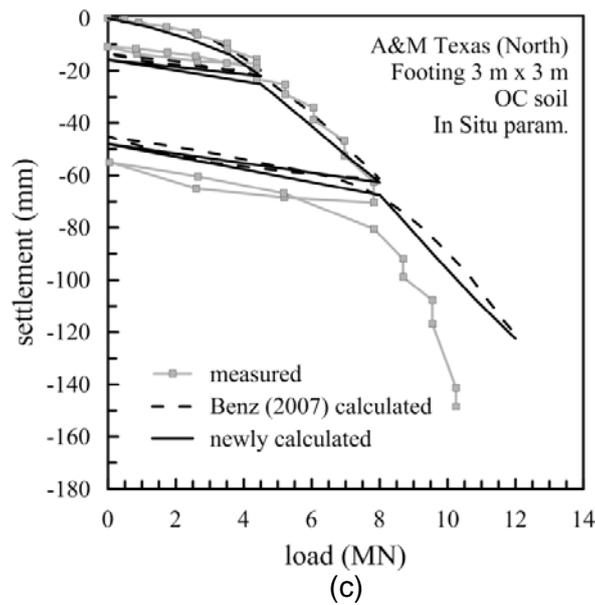


Figure 2. Measured and calculated settlements of A&M Texas 3 m x 3 m North footing: (a) normally consolidated soil (NC), laboratory parameters; (b) overconsolidated soil (OC), laboratory parameters; (c) overconsolidated soil (OC), in situ parameters; measured settlements from Briaud and Gibbens (1997)

Since the value of preloading required for numerical analysis is unknown prior to a load test, it was deemed important to find a way to calibrate the load – settlement curve produced by Plaxis 2D, so that reliable predictions of the foundation soil settlements can be made. The constitutive model to use is the HSSmall one, because it gives the right shape of the nonlinear load – settlement curve for relevant design “strains” up to $s/B = 0.01$. Briaud and Gibbens (1997) use the measured load at this value of “strain” as the allowable load, and the measured load at $s/B = 0.1$ for the soil bearing capacity.

2.2. Calibration of the simulated load – settlement curve

In order to be able to reliably predict shallow foundation settlements in sands by using Plaxis 2D with the HSSmall model, it is necessary to calibrate the simulated load – settlement curve in such a way that the appropriate preloading of the foundation soil is determined. The Mayne et al. (2012) method can be used for this purpose. This method is expressed by the correlation

$$p = 0.585 q_c \sqrt{s/B} \quad (1)$$

where p is the applied footing pressure, and q_c is the CPT cone resistance.

It is assumed in this method that the value of q_c is determined as the average cone resistance of the soil below the footing and down the distance of $2B$. It will be shown in the next Section that the method gives very good matching with measured settlements at 4 locations.

The calibration process is applied by calculating the load – settlement curve from equation (1) and use Plaxis 2D with in situ parameters for the HSSmall model to find the required value of preloading in order to match Plaxis results and the Mayne et al. (2012) curve, particularly at small strains. This process was carried through for the A&M Texas 3 m x 3 m footing North. The preloading of 90 kPa was found to match equation (1), where $q_c = 7.5$ MPa (Mayne et al., 2012). The same value of preloading was used for other footings



at A&M Texas (Fig. 3). Fig. 3 also shows Plaxis curves for normally consolidated foundation soil. It can be seen that the preloading of the soil gives much better settlement predictions than for the normally consolidated soil.

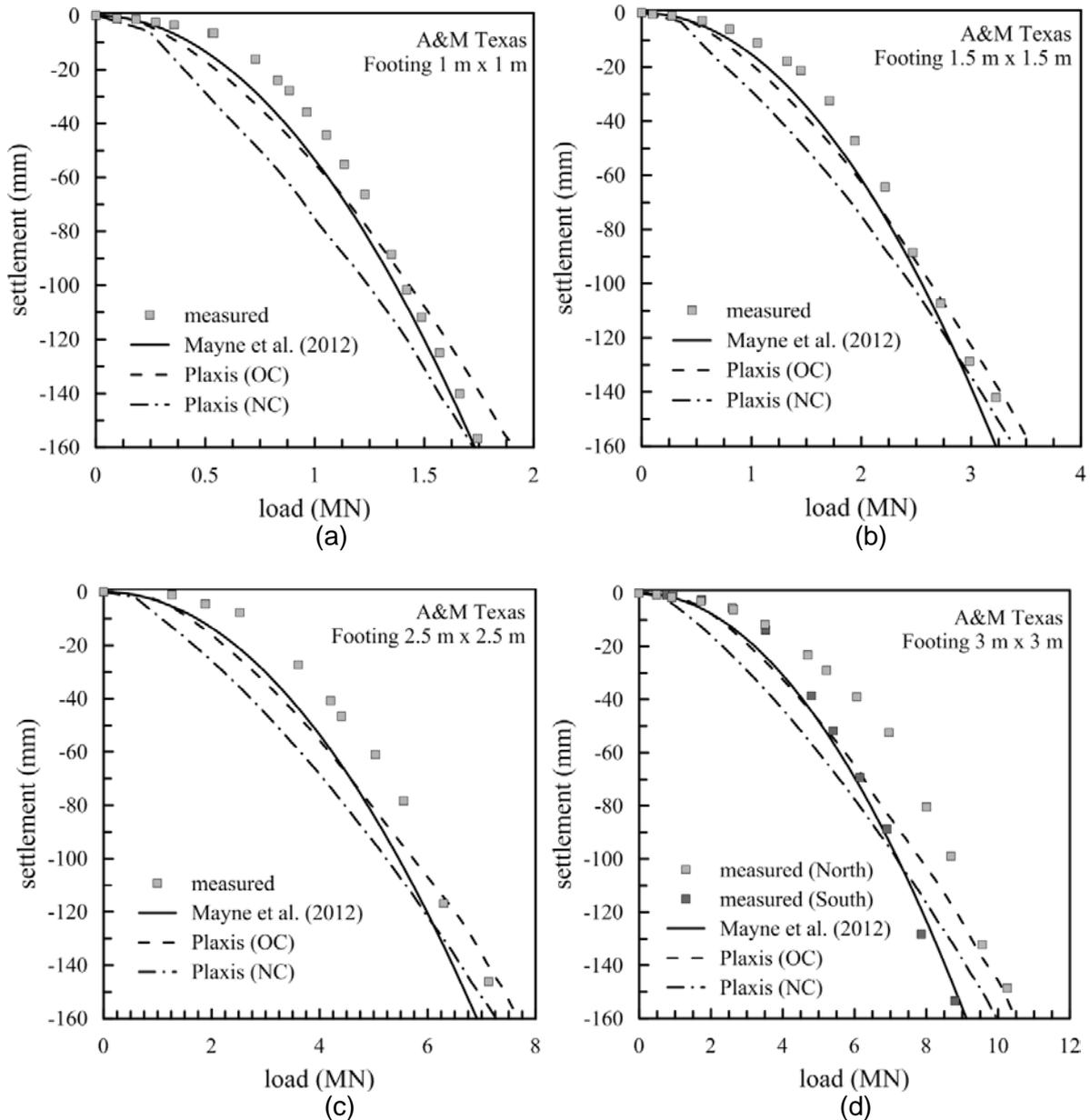


Figure 3. Measured and calculated settlements for all A&M Texas footings; measured settlements from Briaud and Gibbens (1997); calculated by Plaxis 2D for overconsolidated soil through calibration with Mayne et al. (2012); calculated by Plaxis 2D for normally consolidated soil

3. The new direct method for calculation of shallow foundation settlements in sand

3.1. The new method

In this method the “strain” s/B is decomposed into the elastic and plastic components

$$\varepsilon = \varepsilon^e + \varepsilon^p \quad (2)$$

The elastic component is, according to Mayne and Poulos (1999), defined as

$$\varepsilon^e = \left(\frac{s}{B} \right)^c = \frac{pI}{E_0} \quad (3)$$

$$I = \frac{I_h}{\sqrt{\pi/2}} \quad (4)$$

$$I_h = I_G I_F I_E (1 - \nu^2) \quad (5)$$

Mayne and Poulos (1999) considered a flexible circular footing of diameter d , thickness t and modulus of elasticity E_f . The depth of embedment is D_f into the soil of thickness h from the footing base to the bedrock. Young's modulus of the foundation soil is linearly increasing with depth (Gibson, 1967) so that at the footing base it is E_0 , and at depth $zE = E_0 + k_E z$ (Fig. 4).

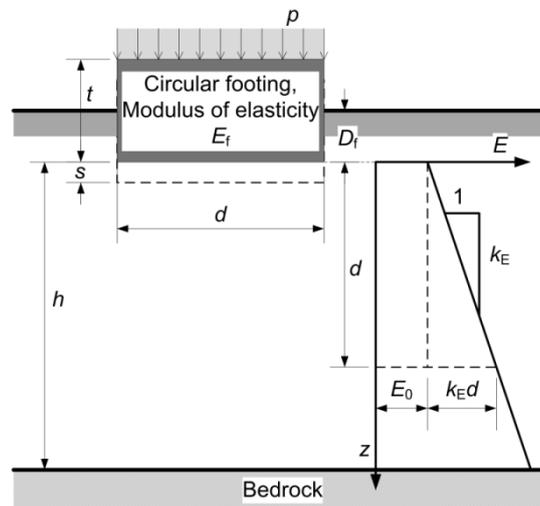


Figure 4. Notations related to the Mayne and Poulos (1999) method for calculation of settlements of a flexible circular footing embedded in the Gibson type of soil (Gibson, 1967)

Let

$$\beta = \frac{E_0}{k_E d} \quad (6)$$

Then the nonhomogeneity factor can be written as



$$I_G \approx \frac{1.6 \frac{h}{d}}{\left(1 + \frac{0.6}{\beta^{0.8}}\right) \left(1 + 1.6 \frac{h}{d}\right)} \quad (7)$$

The stiffness factor of the elastic footing is given by Mayne and Poulos (1999) as

$$I_F = \frac{\pi}{4} + \frac{1}{\frac{1}{1 - \pi/4} + 10 \frac{E_f}{E_0 + 0.5k_E d} \left(\frac{2t}{d}\right)^3} \quad (8)$$

The factor of footing embedment is given by Mayne and Poulos (1999) as

$$I_E \approx 1 - \frac{1}{3.5 \exp(1.22\nu - 0.4) \left(\frac{d}{D_f} + 1.6\right)} \quad (9)$$

The plastic component of equation (2) is, according to Mayne et al. (2012), defined as

$$\varepsilon^p = a \left(\frac{p}{p_L}\right)^b \quad (10)$$

where p_L is the pressure at the „strain“ $s/B = 0.1$ (i.e. the bearing capacity), and a and b are parameters to be determined.

From equations (2) and (10) it can be written

$$\varepsilon^p = \varepsilon - \varepsilon^e = \frac{s}{B} - \frac{pI}{E_0} = a \left(\frac{p}{p_L}\right)^b \quad (11)$$

from which it follows that for $p = p_L$

$$a = 0.1 - \frac{p_L I}{E_0} \quad (12)$$

This gives the expression for the new method for calculation of shallow foundation settlements in sand as

$$\frac{s}{B} = \frac{pI}{E_0} + \left[0.1 - \frac{p_L I}{E_0}\right] \left(\frac{p}{p_L}\right)^b \quad (13)$$

Parameters b and p_L are determined by the least squares method for measured settlements during each of 16 load tests performed at 4 locations. Then, parameter b is determined as the average value for the 16 footings, and a correlation is set between p_L for all footings and the CPT cone resistance at 4 locations.



3.2. Data base of 16 load tests at four locations

The four locations chosen for the determination of parameters b and ρ_L are listed in Table 2. In total 16 footings were load tested at these locations. It has to be noted that Mayne et al. (2012) define the slope of applied footing pressure vs. $(s/B)^{0.5}$ and list its value for each of 13 analyzed locations. The four locations in Table 2 cover the whole range of slope values for 12 out of the 13 locations. In the case of A&M Texas location the value of the slope is the greatest, and in the case of Perth location it is the smallest out of 12

locations, even though values of the corresponding CPT cone resistance do not have the same trend. All footings are made of reinforced concrete (RC). For all footings measured settlements during load tests are available in the listed literature.

Table 2. Testing sites, footing equivalent square side and references (from Mayne et al., 2012)

Name of site	Location	Type of sand	Footing side	Depth of embedment, D_f (m)	Reference
A&M Texas	USA	Pleistocene sand	5 square RC: $B = 1.0; 1.5; 2.5$ and 3.0 m	0.76	Briaud and Gibbens (1997)
Perth	Australia	Siliceous dune sand	4 square RC: $B = 0.67; 1.0$ and 1.5 m	0.5 to 1.0	Lehane et al. (2008)
Fittja	Sweden	Glaciofluvial sand	3 rectangular RC: $B = 0.6; 1.7$ and 2.4 m	0.4 to 1.6	Bergdahl et al. (1985)
Kolbytteimon	Sweden	Glaciofluvial sand	4 rectangular RC: $B = 0.6; 1.2; 1.7$ and 2.4 m	0.4 to 1.6	Bergdahl et al. (1985)

3.3. Determination of parameters for the new method

From the least squares method 16 values were obtained for each of parameters b and ρ_L (Table 3). One example of measured data and the fitted curve for each location is shown in Fig. 5.

Table 3. Values of parameters ρ_L and b for all 16 footings

Footing dimensions	ρ_L (kPa)	b
A&M Texas		
1m x 1m	1463	2.56
1.5m x 1.5m	1435	2.32
2.5m x 2.5m	1402	2.62
3m x 3m (north)	1474	2.71
3m x 3m (south)	1266	2.44
Perth		
0.67m x 0.67m x 1m	635.9	1.58
1m x 1m x 0.5m	533.6	1.85



1m x 1m x 1m	533.5	1.85
1.5m x 1.5m x 1m	380.7	2.42
Fittja		
0.55m x 0.65m	635.1	1.99
1.6m x 1.8m	750.7	1.66
2.3m x 2.5m	651.9	2.25
Kolbyttemon		
0.55m x 0.65m	1731	2.08
1.1m x 1.3m	2061	2.02
1.6m x 1.8m	1908	1.99
2.3m x 2.5m	1869	1.97
Average		2.14

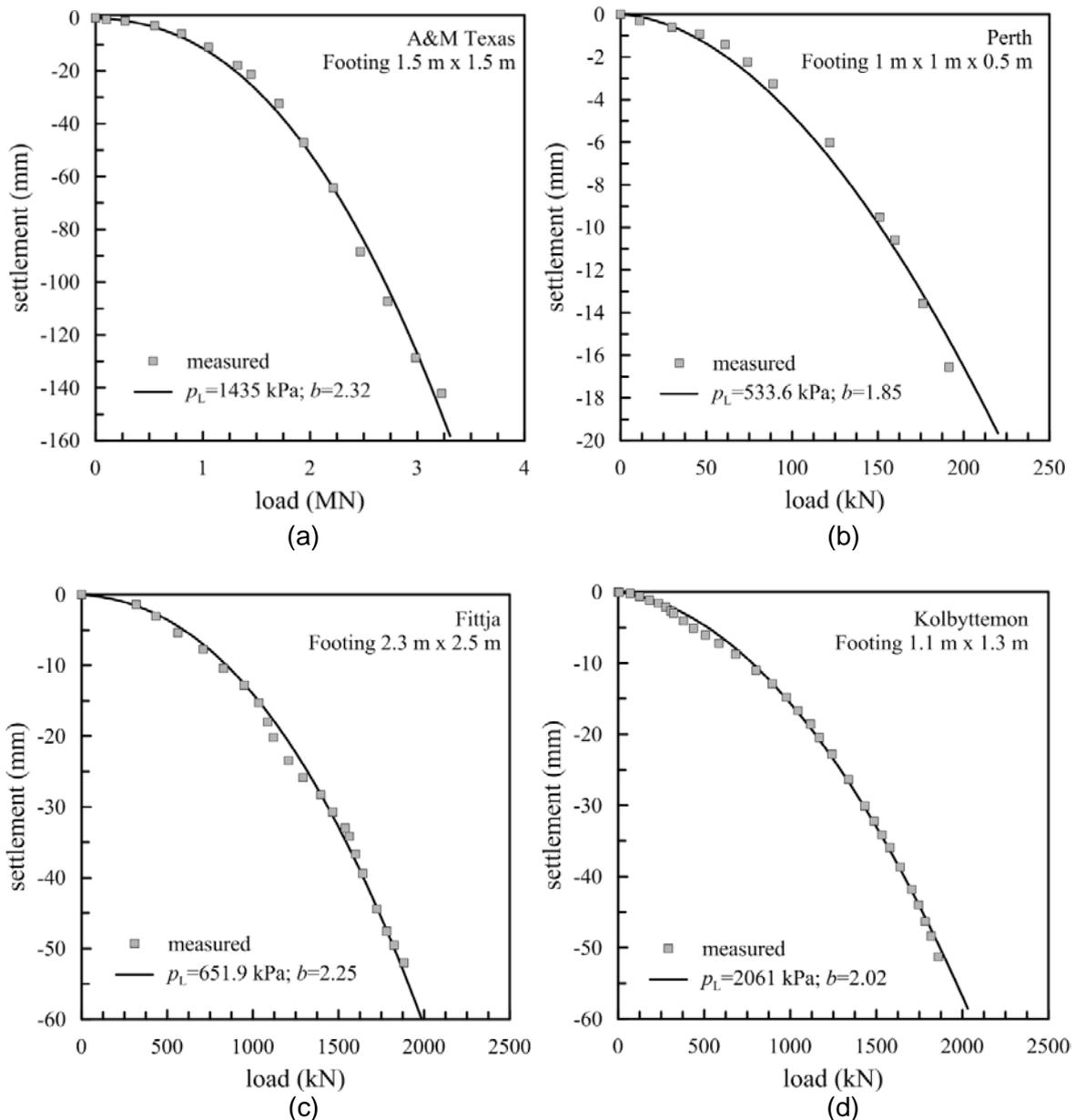


Figure 5. Measured data and fitted curve for one footing from each location: p_L and b from Table 3



The average value is $b = 2.14$. This value is very close to the exponent used by Mayne et al. (2012), $b = 2$. Fig. 6 shows 16 values of p_L and the corresponding q_c for the four respective locations.

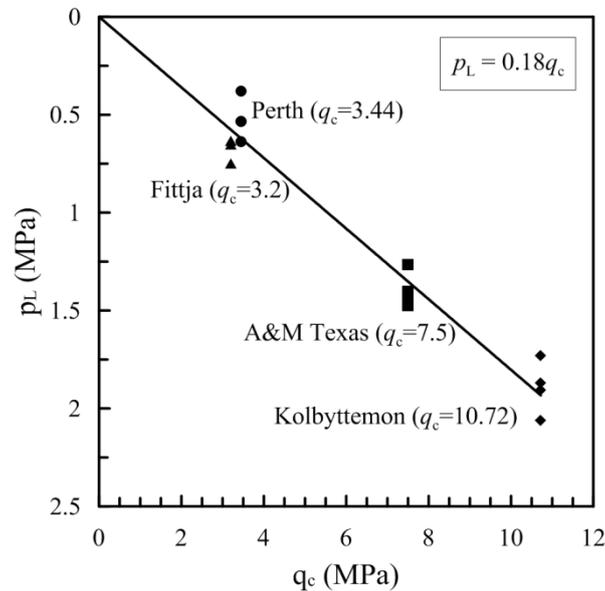


Figure 6. p_L and q_c for all 16 footings (q_c values taken from Mayne et al., 2012)

The best fit line in Figure 6 gives

$$p_L = 0.18q_c \quad (14)$$

This correlation is same as in Mayne et al. (2012).

It is, thus, to be expected that the new method gives very similar results to those obtained by equation (1) proposed by Mayne et al. (2012). This is evidenced in Fig. 7 for the four locations. The matching of measured normalized settlement values, s/B and those calculated by the two methods is very good. The calculated curves slightly overpredict measured settlements for A&M Texas, and slightly underpredict them for Perth. The reason for this is that these two locations have the largest and the smallest value of the slope of applied footing pressure vs. $(s/B)^{0.5}$ respectively, as stated above.

The final form of the equation for the new method is given by

$$\frac{s}{B} = \frac{pI}{E_0} + \left[0.1 - \frac{0.18q_c I}{E_0} \right] \left(\frac{p}{0.18q_c} \right)^{2.14} \quad (15)$$

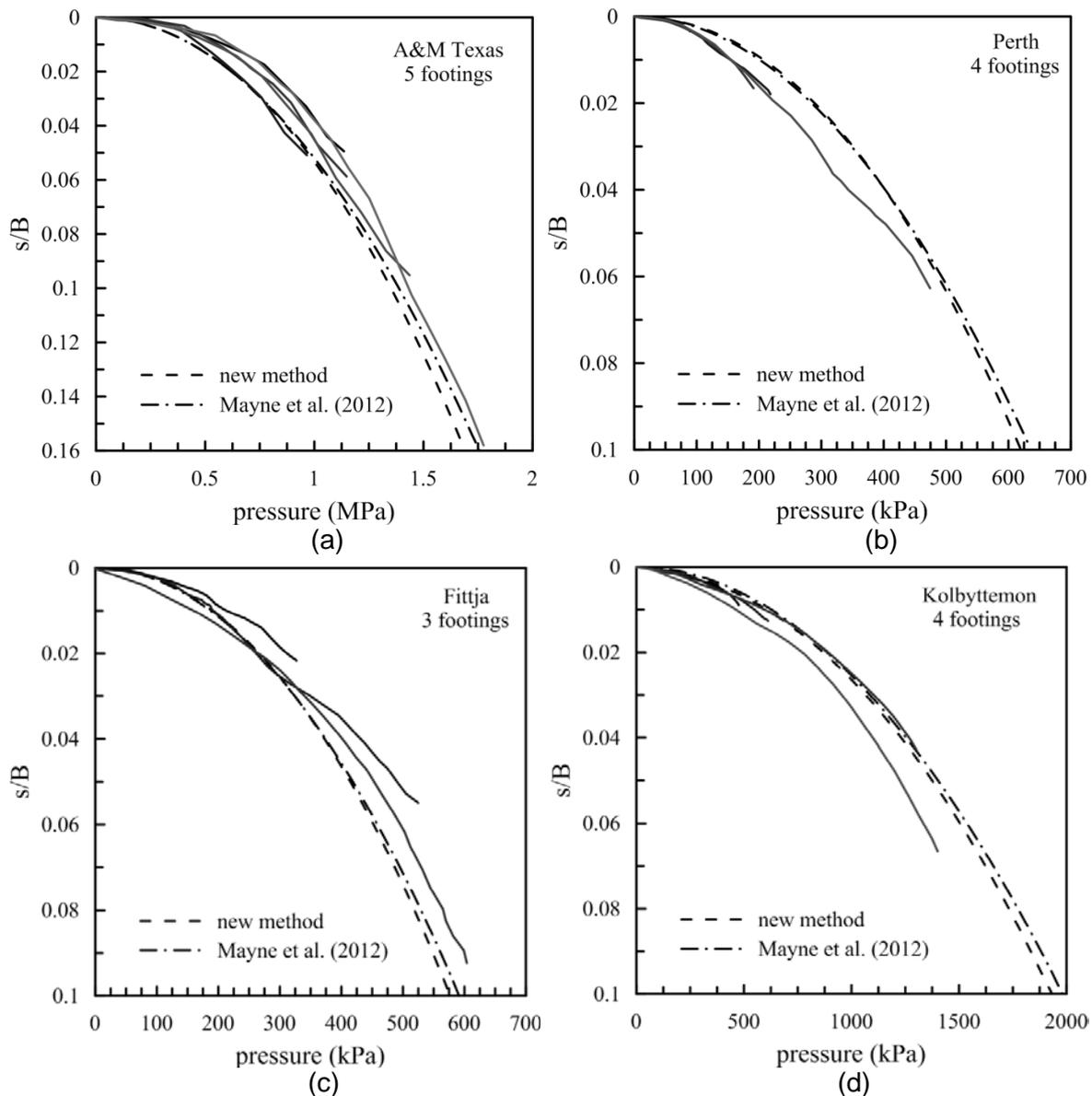


Figure 7. Normalized settlement vs. applied footing pressure for the four locations

Another feature related to measured data is illustrated in Fig. 7. Fellenius and Altaee (1994) showed that when data measured during load tests on sand are plotted as s/B vs. the applied footing pressure, the resulting curves for square footings of different sizes ($B = 0.25, 0.50, 0.75$ and 1.0 m) almost coincide. This was also shown by Briaud and Gibbens (1997) for the 5 A&M Texas footings, which is illustrated in Fig. 7a. It is shown here that the same holds true for the other three analyzed test sites, as can be seen in Figs 7b to 7d. This means that the sand bearing capacity can be determined from the simple correlation given by equation (14), and that it does not depend on the footing width. This is contrary to the basic Terzaghi expression for the sand bearing capacity for a strip foundation on the soil surface, $p_L = 0.5\gamma BN_\gamma$ where γ is the soil unit weight and N_γ is the bearing capacity factor which depends on the angle of friction ϕ' . Briaud and Gibbens interpret this as either the soil bearing capacity does not depend on the footing width, or the bearing capacity factor N_γ , besides on ϕ' , also depends on the foundation width.



3.4. Very small strains

The important difference between the new method and the Mayne et al. (2012) method is in the range of very small strains. This is illustrated in Fig. 8, where the applied footing pressure is presented on the logarithmic scale. Fig. 8 shows a distinct and correct deviation of the curve calculated by the new method from the abscissa, at the slope corresponding to E_0 . The tangent to the curve calculated by (Mayne et al., 2012) is the abscissa itself, indicating infinite soil stiffness at very small strains. The soil behavior at very small strains in the new method is an addition to the Mayne et al. (2012) method, which enables seismic analyses of shallow foundations.

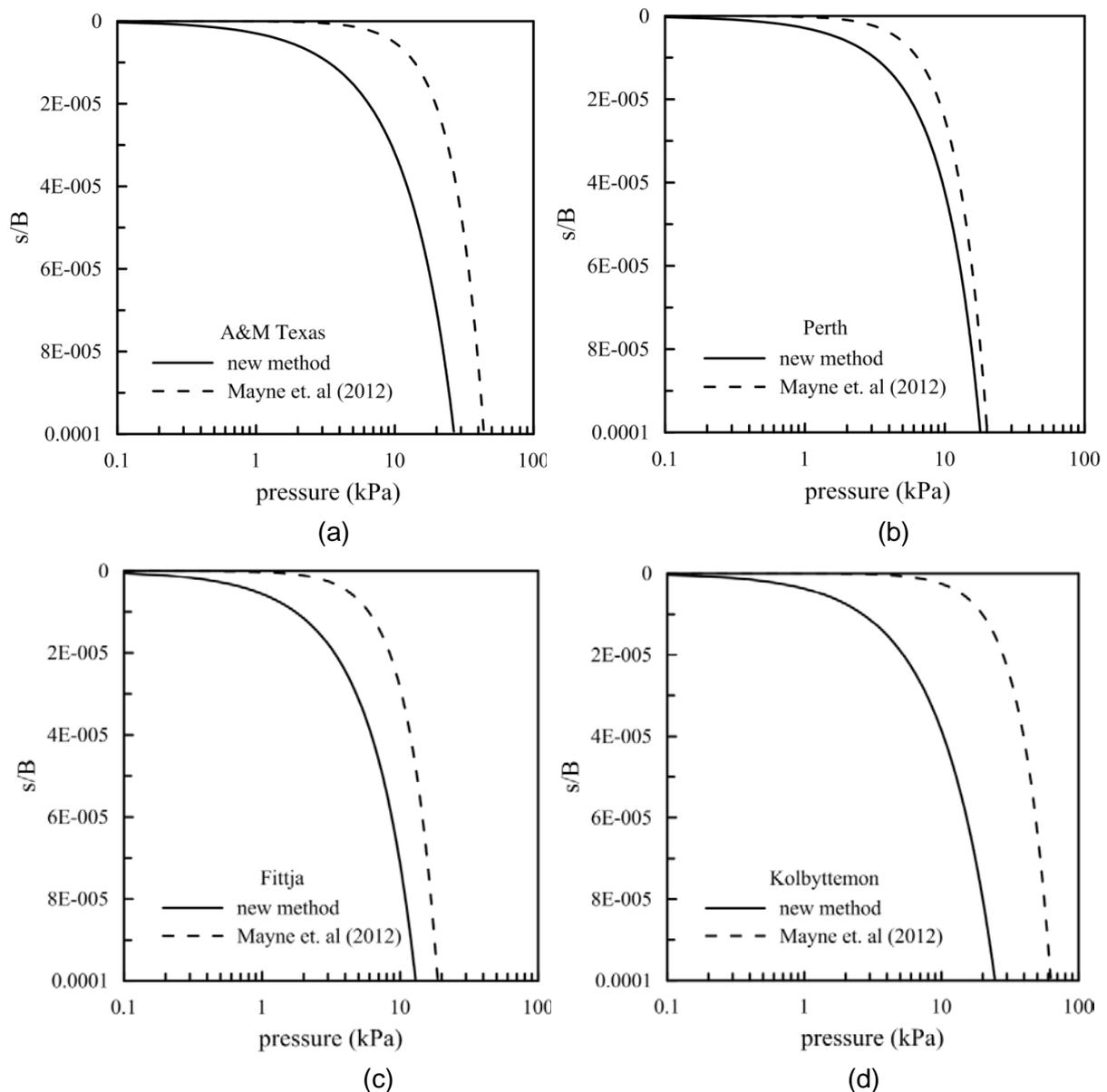


Figure 8. Normalized settlement vs. applied footing pressure at very small strains for the four locations



4. Conclusions

According to Benz (2007), the Hardening Soil Small constitutive relationship incorporated in Plaxis 2D matches very well settlements measured in sand during a load test of a square footing 3 m in size, performed at the University A&M Texas (Briaud and Gibbens, 1997). However, this good matching is obtained after the foundation soil was preloaded and unloaded prior to the load test in the numerical analysis, in order to make it overconsolidated. Imposing the a priori unknown preloading to match calculated with measured settlements is a substantial obstacle in using numerical modeling to predict shallow foundation settlements in sand in practice. On the other hand, empirical relationships (Akbas and Kulhawy, 2009; Mayne et al., 2012) cannot be used for prediction of shallow foundation settlements in layered soils, including sands. The same is true for large shallow foundations on relatively thin sand layers. In these cases settlement predictions based on numerical modeling are advantageous, but only if properly calibrated.

It is shown in the paper that numerical modeling of shallow foundation settlements can be successfully performed by Plaxis 2D and the HSSmall constitutive relationship if the simulated load – settlement curve is calibrated, in terms of the preloading value, against the curve calculated by the Mayne et al. (2012) method. Very good results are obtained when this procedure is used for 5 square footings ranging in size from 1 m to 3 m, tested at A&M Texas (Briaud and Gibbens, 1997).

The Mayne et al. (2012) method is very simple and it uses only one parameter, the CPT cone resistance q_c . However, it cannot predict soil settlements at very small strains, which is important for seismic analyses of shallow foundations, because in this range of strains the soil stiffness given by Mayne et al. (2012) is infinite. A new direct method for calculation of shallow foundation settlements in sand, which eliminates this problem, is presented in the paper. It was developed along the lines of the Mayne et al. (2012) method, based on the Mayne and Poulos (1999) approach, with an addition for elastic (very small) strains. This method gives the correct soil stiffness at very small strains, which corresponds to the initial Young's modulus E_0 , easily obtainable from in situ shear wave velocity measurements.

The new method was developed from the data base of 16 load tests at 4 locations, which cover the relevant range of 12 out of 13 locations analyzed by Mayne et al. (2012). Thus, one of the two unknown parameters from the new method is obtained in the same form of correlation as in Mayne et al. (2012), and the other has a very similar value to the one by these authors. This is the reason why the two methods give very similar results for the 4 analyzed locations, except at very small strains, where the results differ substantially.

It is also reiterated from Mayne et al. (2012) that the sand bearing capacity can be obtained from a simple correlation with the CPT cone resistance, since Fellenius and Altaee (1994) and Briaud and Gibbens (1997) showed that measured data from load tests on square footings of different sizes on sands almost coincide when plotted in s/B vs. applied footing pressure diagram. The test location in Briaud and Gibbens (1997) is one out of four analyzed here, and it is shown that the same holds true for the other three locations. This indicates that the sand bearing capacity does not depend on the footing width, as previously noted by Briaud and Gibbens (1997).

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