Received: 16.04.2024.
Accepted: 17.06.2024.

e-ZBORNIK 27/2024

Professional paper

Electronic collection of papers of the Faculty of Civil Engineering

https://doi.org/10.47960/2232-9080.2024.27.14.59

ISSN 2232-9080

THE JACKSON AND MORELAND ALIGNMENT CHARTS

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Abstract: The paper explains the method of developing the Jackson and Moreland alignment charts, which are used to determine the effective buckling length of load-bearing elements subjected to compressive loading. The result of the paper is applicable in the fields of calculation and dimensioning of load-bearing structures.

Key words: alignment chart, effective buckling length, horizontally immovable structures, horizontally movable structures

Jacksonovi i Morelandovi nomogrami

Sažetak: : U radu je objašnjena metoda izrade Jacksonovih i Morelandovih nomograma koji služe za određivanje efektivne duljine izvijanja nosivih elemenata opterećenih na tlak. Rezultat rada je primjenjiv u oblastima za proračun i dimenzioniranje nosivih konstrukcija.

Ključne riječi: nomogram, efektivna duljina izvijanja, horizontalno nepomične konstrukcije, horizontalno pomične konstrukcije

1. INTRODUCTION

Buckling of load-bearing elements subjected to compressive loading is a problem when dimensioning structures. The occurrence of buckling, as an unintended consequence during the construction and operation of structures, is treated in the rules and standards for the dimensioning of load-bearing structures in construction. Buckling is especially manifested in slender and high elements subjected to compressive loading, and therefore additional calculations are required.

Slender compression elements of frame, lattice and similar structures, usually already at the beginning of loading are not straight, as specified by the project, but are bended. Initial curves can be of geometric or static origin. Geometric curves (deformations), which are the result of incorrect execution or some other cause, are assumed to follow the form of buckling of centrically stressed elements. Static curves, which are the result of the action of the bending moment along the axis of the element, depend on the change of static values along the length of the element, on the method of connecting the element (boundary conditions), presence of transverse load and slenderness of the element. Deflections, which are a consequence of these actions, can be considerable and must not be ignored. The stability of the structure and of the element must be observed on a deformed system. For part of the calculation for the stability of the observed load-bearing element, it is necessary to determine the coefficient of effective buckling length, or the effective buckling length of the same element. To determine the above coefficient, it is necessary to calculate the values of fixity on the edges of the slender compression element.

An alignment chart is a graphic representation of the dependence of two or more quantities, which makes it possible to determine the unknown values, that is, alignment chart is a graphical calculator. It consists of several scales placed so that the dependence of a quantity on one or more other quantities can be obtained by drawing a line over the graph or by some other simple combination of points on the scales that correspond to these quantities.[2]

The Jackson and Moreland alignment charts are precise graphical representations of buckling length coefficient solutions for possible values of fixity at the ends of an element subjected to compression loading. The same alignment charts are used when determining the effective buckling length of a column in load-bearing structures. The first type of alignment chart is used to determine the coefficient of effective buckling length for horizontally immovable structures. The second type of alignment chart is used to determine the coefficient of effective buckling length for horizontally movable structures. The Jackson and Moreland alignment charts are applicable in the fields of calculation and dimensioning of load-bearing structures.

This paper deals with the structure of the algorithm for calculation of relevant values for the formation of Jackson and Moreland alignment charts. MS Office Excel software packages and Python programming language were used for the calculation procedure and graphic presentation. The paper is designed so that the following chapter presents the theoretical foundations and assumptions necessary for understanding the approach to developing Jackson and Moreland alignment charts. The third chapter describes the structure of the loop program necessary for determining ordinates in alignment charts. The concluding, fourth chapter, presents the results as the Jackson and Moreland alignment charts. The developed Jackson and Moreland alignment charts have a denser resolution of scales, which makes it possible to determine more accurately and precisely the required values, which are important for further calculation in the dimensioning of slender elements subjected to compressive loading.

2. EQUATIONS, DIAGRAMS OF EQUATIONS AND ASSUMPTIONS

2.1 Equations for horizontally immovable and movable structures

The equation that describes the correlation between the coefficient of effective buckling length K (β) and the coefficients of fixity at the column ends GA (KA) and GB (KB) for horizontally immovable structures is given below [3]:

$$\frac{G_A G_B}{4}\left(\frac{\pi}{K}\right)^2 + \left(\frac{G_A+G_B}{2}\right)\left[1-\frac{(\pi/K)}{\tan(\pi/K)}\right] + \frac{2\tan(\pi/(2K))}{(\pi/K)} - 1 = 0 \tag{1}$$

or:

$$\frac{K_A K_B}{4}\left(\frac{\pi}{\beta}\right)^2 + \left(\frac{K_A+K_B}{2}\right)\left[1-\frac{(\pi/\beta)}{\tan(\pi/\beta)}\right] + \frac{2\tan(\pi/(2\beta))}{(\pi/\beta)} - 1 = 0 \tag{2}$$

The equation that describes the correlation between the coefficient of effective buckling length K (β) and the coefficients of fixity at the column ends GA (KA) and GB (KB) for horizontally movable structures is given below:

$$\frac{G_A G_B(\pi/K)^2-36}{6(G_A+G_B)} - \frac{(\pi/K)}{\tan(\pi/K)} = 0 \tag{3}$$

or:

$$\frac{K_A K_B(\pi/\beta)^2-36}{6(K_A+K_B)} - \frac{(\pi/\beta)}{\tan(\pi/\beta)} = 0 \tag{4}$$

where in equations (1), (2), (3) and (4):
G_A (K_A) - is the column stiffness coefficient at the top of the column at node A,
G_B (K_B) - is the column stiffness coefficient at the bottom of the column at node B,
K (β) - is the coefficient of effective buckling length of the column from node A to node B.

2.2 Diagrams of equations for horizontally immovable and movable structures

It can be observed on the diagrams that the equations have many zero points or solutions. Therefore, the problem of searching for the desired result was solved by applying the bisection method in the intervals containing the required value. The bisection method is applicable for situations of horizontally immovable and movable structures.

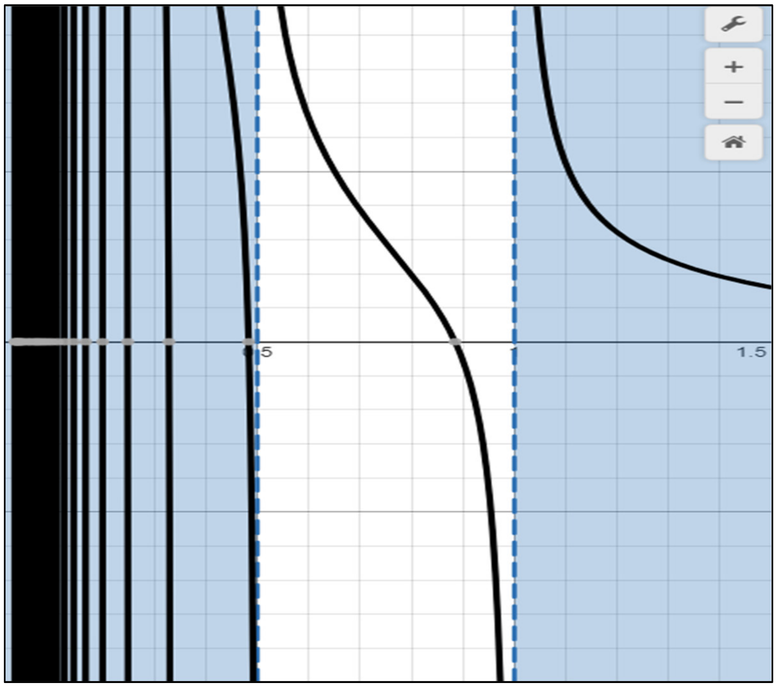

Figure 1. Diagram with equation (1) or (2) for horizontally immovable structures (bisection area is in interval [0.5,1.0] - unshaded area)

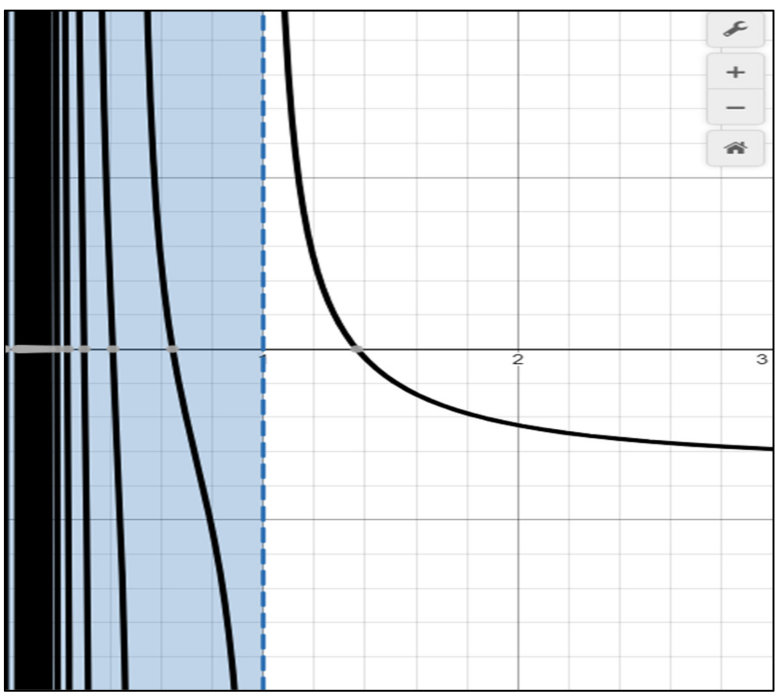

Figure 2. Diagram with equation (3) or (4) for horizontally movable structures (bisection area is in interval [1.0, 〖10〗 ^100] - unshaded area)

2.3 Assumptions

The assumptions for calculation of the ordinates of characteristic values on the alignment charts are given below:

- The number infinity ∞ from the previous alignment charts is rounded to the number 10^{100}.
    - For easier calculation and understanding of alignment chart design.
    - The number 10^{100} simulates well the number infinity ∞ in the results relevant to alignment chart design. When increasing the exponent number 100 no difference in results is observed. The exponent number 100 could be even smaller, and also no difference would be observed in the results relevant for forming the alignment chart.
    - Using the bisection method, to reach the number with 16 decimal places from the number 10^{100}, it takes $= \log_2 10^{(100+16)} = +385 \approx 400$ steps, which is not a large number of steps for today's computers.
    - Numbers greater than 10^{100} will not be entered into the calculation.
- Each slanted and straight line drawn on the alignment charts gives the exact result of the required value.
- Axes for fixity quantities G_A and G_B (K_A and K_B) are equal and have equal ordinates for characteristic values.

3. LOOP CALCULATION STRUCTURE

The structure of an algorithm for calculating the relevant ordinate values for alignment chart cases of horizontally immovable and horizontally movable structures is explained below. The same algorithm consists of a series of loops as follows:

- loop (P1) for numerical calculation of K (β) for input values G_A (K_A) and G_B (K_B),
- loop (P2) for numerical calculation of $G_A = G_B$ ($K_A = K_B$) for the input value K (β),
- loop for numerical calculation of the ordinate y for the value G_A (K_A) and G_B (K_B),
- loop for numerical calculation of the ordinate y for the value K (β).

3.1 The K (β) value search loop (P1) for input values GA (KA) and GB (KB)

The K (β) value search loop for input values of stiffening coefficients G_A (K_A) and G_B (K_B) searches for the result using the bisection method in intervals:

- for horizontally immovable structures $K \in [0.5, 1.0]$ ($\beta \in [0.5, 1.0]$),
- for horizontally movable structures $K \in [1.0, 10^{100}]$ ($\beta \in [1.0, 10^{100}]$).

In this loop, the values of K (β) are calculated in equations (1) or (2) for horizontally immovable structures and in equations (3) or (4) for horizontally movable structures. This loop calculates the value of K (β) for positive input values greater than zero.

In order to avoid numerical errors that occur in the case of small input values less than $\sim 10^{-10}$, in loop (P1) it is recommended to round the input numbers that are positive and tend to zero ($G_A \to 0$ ($K_A \to 0$) and $G_B \to 0$ ($K_B \to 0$)), to the arbitrary and tested number 10^{-7}.

3.2 The GA = GB (KA = KB) value search loop (P2) for the input value K (β)

A simplified equation for horizontally immovable structures with two unknown quantities $G_{A,B} = G_A = G_B$ ($K_{A,B} = K_A = K_B$) and K (β), is given below:

$$\frac{{G_{A,B}}^2}{4}\left(\frac{\pi}{K}\right)^2 + G_{A,B}\left[1-\frac{(\pi/K)}{\tan(\pi/K)}\right] + \frac{2\tan(\pi/(2K))}{(\pi/K)} - 1 = 0 \quad (5)$$

or:

$$\frac{{K_{A,B}}^2}{4}\left(\frac{\pi}{\beta}\right)^2 + K_{A,B}\left[1-\frac{(\pi/\beta)}{\tan(\pi/\beta)}\right] + \frac{2\tan(\pi/(2\beta))}{(\pi/\beta)} - 1 = 0 \quad (6)$$

A simplified equation for horizontally movable structures with two unknown quantities $G_{A,B} = G_A = G_B$ ($K_{A,B} = K_A = K_B$) and K (β), is given below:

$$\frac{{G_{A,B}}^2(\pi/K)^2-36}{12\cdot G_{A,B}} - \frac{(\pi/K)}{\tan(\pi/K)} = 0 \quad (7)$$

or:

$$\frac{{K_{A,B}}^2(\pi/\beta)^2-36}{12\cdot K_{A,B}} - \frac{(\pi/\beta)}{\tan(\pi/\beta)} = 0 \quad (8)$$

where (in equations (5), (6), (7) and (8)):

$G_{A,B}$ ($K_{A,B}$) - is the column stiffness coefficient at the top and bottom of the observed column (at node A and at node B),

K (β) - is the coefficient of effective buckling length of the column from node A to node B.

The loop for searching for the value $G_{A,B} = G_A = G_B$ ($K_{A,B} = K_A = K_B$) for the input value K (β) searches for the result of equal values G_A and G_B (K_A and K_B) using the bisection method in intervals:

- for horizontally immovable structures $G_{A,B} \in [0.0, 10^{100}]$ ($K_{A,B} \in [0.0, 10^{100}]$),
- for horizontally movable structures $G_{A,B} \in [0.0, 10^{100}]$ ($K_{A,B} \in [0.0, 10^{100}]$).

In this loop, the values of $G_{A,B}$ ($K_{A,B}$) are calculated in simplified equations with two unknown quantities, in equations (5) or (6) for horizontally immovable structures and in equations (7) or (8) for horizontally movable structures.

3.3 The ordinate y search loop for the value GA (KA) and GB (KB)

The alignment chart drawing area is in a two-dimensional coordinate system within the limits of the abscissa $x \in [0.0, 1.0]$ and the ordinate $y \in [0.0, 1.0]$. The ordinate values are calculated by loops, while the abscissa values for certain quantities are given below:

- for quantity G_A (K_A) $x = 0.0$
- for quantities K (β) $x = 0.5$
- for quantity G_B (K_B) $x = 1.0$

The ordinate y search loop for the values G_A (K_A) and G_B (K_B) is based on the principle that the number infinity ∞ is rounded to the number 10^{100}. The advantage of this principle is that by calculating the value of K (β) by loop (P1) with the input data $G_A = 0.0$ and $G_B = 10^{100}$ (or $K_A = 0.0$ and $K_B = 10^{100}$) in equation (1) or (2) and in equation (3) or (4), in addition to the

K (β) value, one obtains the value of the ordinate y on the alignment charts, which is equal to $y = 0.5000$ in the first step (Figure 3). In the first step after loop (P1), the value of equal quantities G_A and G_B (K_A and K_B) ($G_A = G_B = K_A = K_B$) is calculated by loop (P2) and this calculation reduces the ordinate search area for the next step of the loop, which, like the first step, is a combination of loop (P1) and loop (P2).

The images below show graphically the first several steps when searching for the ordinate y for the situation of horizontally immovable structures for the value G_A=G_B=0.25 (or K_A=K_B=0.25). With this loop, further calculation eventually results in the solution of the ordinate, which is equal to y=0.2900174850495052.

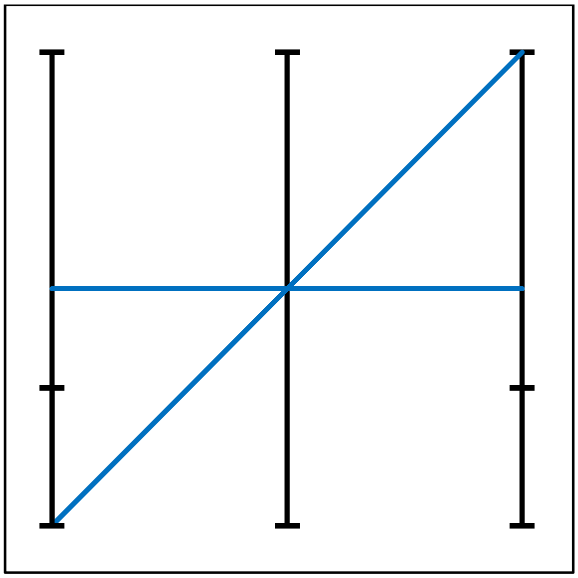

Figure 3. The first step of the loop, in which the ordinate is equal to y=0.5000, within the ordinate limits y_lower=0.0000 and y_upper=1.0000

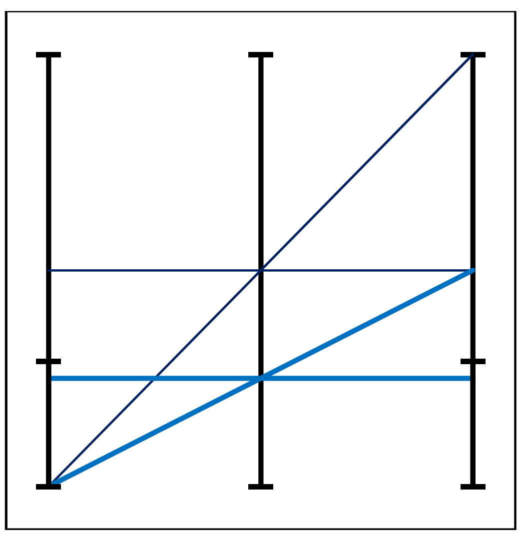

Figure 4. The second step of the loop, in which the ordinate is equal to y=0.2500, within the ordinate limits y_lower=0.0000 and y_upper=0.5000

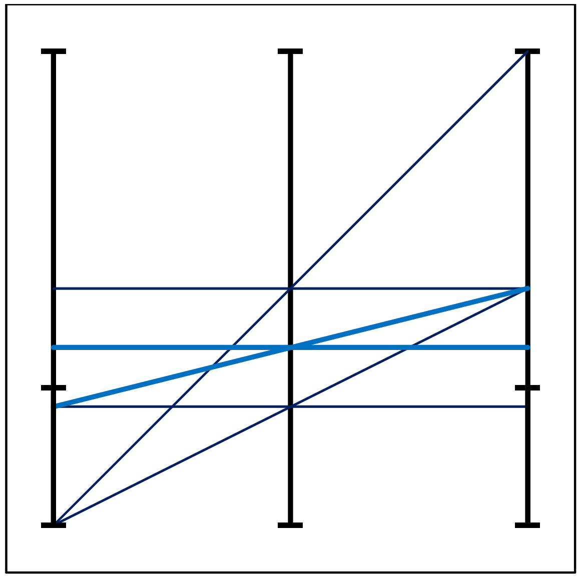

Figure 5. The third step of the loop, in which the ordinate is equal to y=0.3750, within the ordinate limits y_lower=0.2500 and y_upper=0.5000

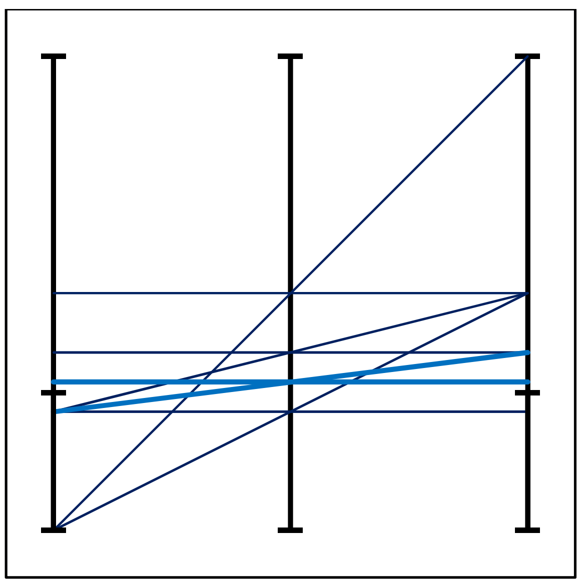

Figure 6. The fourth step of the loop, in which the ordinate is equal to y=0.3125, within the ordinate limits y_lower=0.2500 and y_upper=0.3750

3.4 The ordinate y search loop for the value of K (β)

The ordinate y search loop for the values of K (β) is based on the principle that the number infinity ∞ is rounded to the number 〖10〗^100. The advantage of this principle is that by calculating the value of K (β) by loop (P1) with the input data G_A=0.0 and G_B= 〖10〗 ^100 (or K_A=0.0 and K_B= 〖10〗 ^100) in equation (1) or (2) and in equation (3) or (4), in addition to the value of K (β), one obtains the value of the ordinate y on the alignment charts, which is equal to y=0.5000 in the first step (Figure 7). In the first step after loop (P1), the value of equal quantities GA and GB (KA and KB) (G_A=G_B=K_A=K_B) is calculated by loop (P2) and this calculation reduces the ordinate search area for the next step of the loop, which, like the first step, is a combination of loop (P1) and loop (P2).

The figures below show graphically the first several steps when searching for the ordinate y for the situation of horizontally immovable structures for the value of the coefficient of effective buckling length K=0.90 (or β=0.90). With this loop, further calculation eventually results in the solution of the ordinate, which is equal to y=0.8644423727655408.

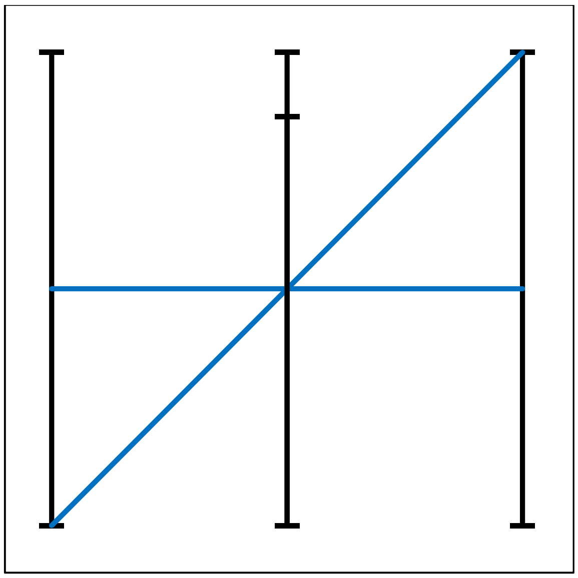

Figure 7. The first step of the loop, in which the ordinate is equal to y=0.5000, within the ordinate limits y_lower=0.0000 and y_upper=1.0000

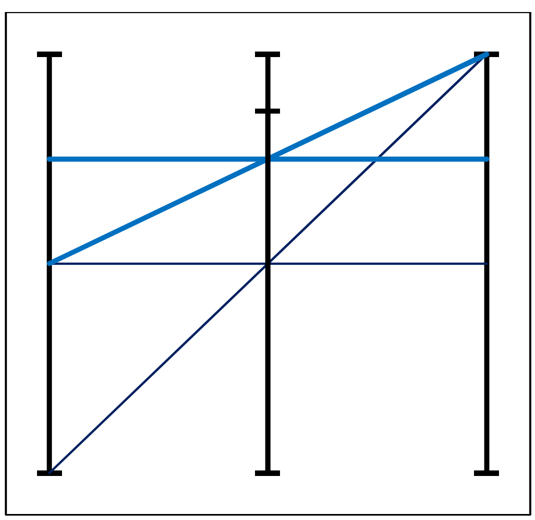

Figure 8. The second step of the loop, in which the ordinate is equal to y=0.7500, within the ordinate limits y_lower=0.5000 and y_upper=1.0000

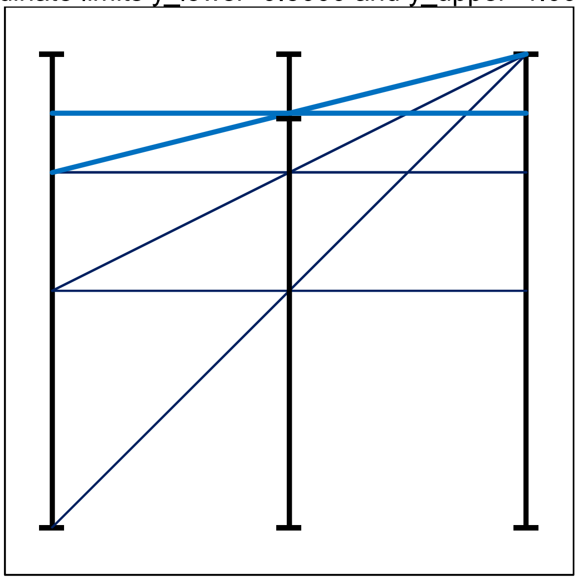

Figure 9. The third step of the loop, in which the ordinate is equal to y=0.8750, within the ordinate limits y_lower=0.7500 and y_upper=1.0000

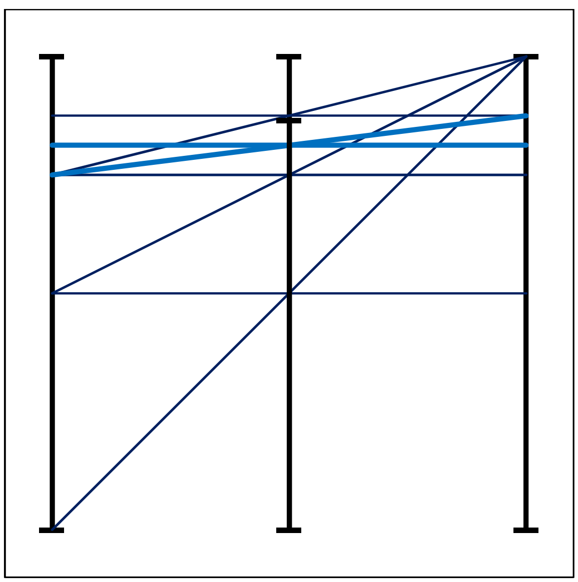

Figure 10. The fourth step of the loop, in which the ordinate is equal to y=0.8125, within the ordinate limits y_lower=0.7500 and y_upper=0.8750

4. THE JACKSON AND MORELAND ALIGNMENT CHARTS

The Jackson and Moreland alignment charts are graphic calculators and are designed in the Excel program of the Microsoft Office package. The Jackson and Moreland alignment charts are given below. Figure 11 shows the Jackson and Moreland alignment chart for determining the coefficient of effective buckling length of horizontally immovable structures.

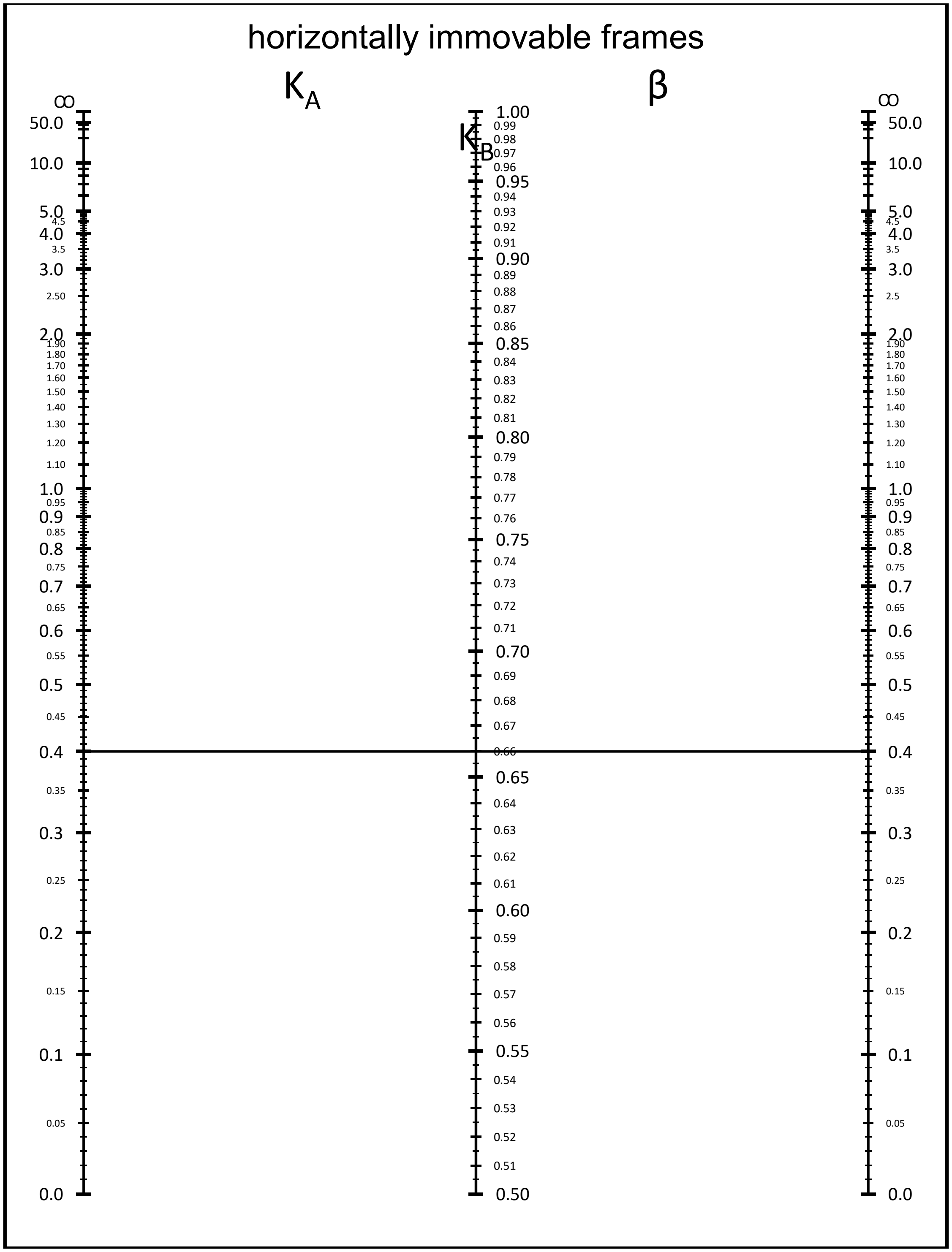

Figure 11. Alignment chart for horizontally immovable structures

Figure 12 shows the Jackson and Moreland alignment chart for determining the coefficient of effective buckling length of horizontally movable structures.

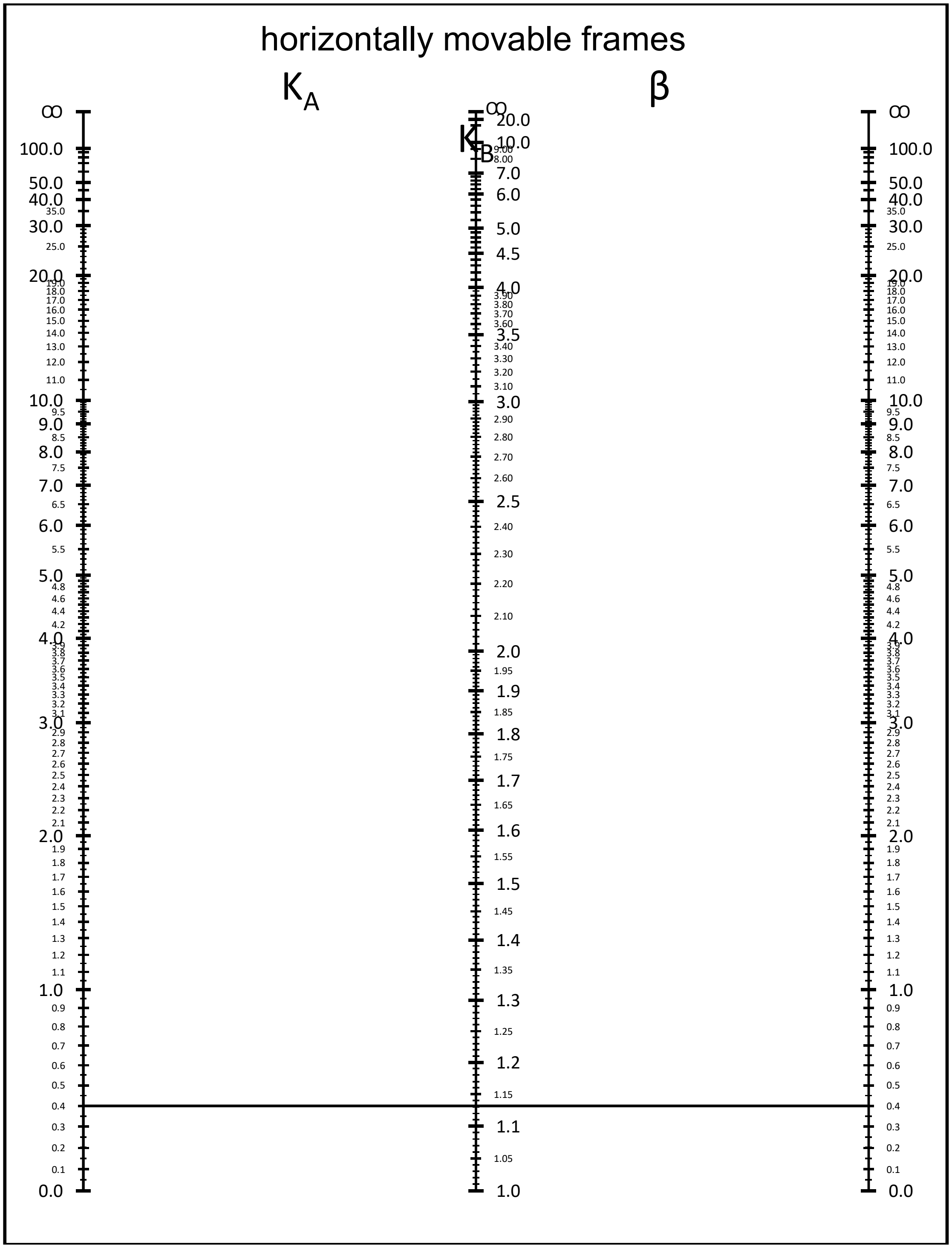

Figure 12. Alignment chart for horizontally movable structures

The previously presented Jackson and Moreland nomograms as graphing calculators consist of three vertical scales for a given size as follows:

- G_A (K_A) of the coefficient of reinforcement of the pressure element on the edge,
- K (β) effective buckling length coefficient, obtained by drawing a line between known points on the G_A (K_A) and G_B (K_B) scales,
- G_B (K_B) the reinforcement coefficient of the observed pressure element on the opposite edge.

On the Jackson and Moreland nomograms, there is a horizontal line as a limit for the inadvisable smaller input values of the G_A (K_A) and G_B (K_B) scales fixtures. The bracing coefficients should not be less than $G_A = G_B = K_A = K_B = 0.40$. Also, if necessary, it is possible to determine one of the bracing coefficients on the edge of the pressure element G_A (K_A) or G_B (K_B) if the calculation required a certain size of the effective buckling length coefficient K (β) and if one of the bracing coefficients on the opposite edge is known of the observed compressively loaded load-bearing element.

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